Cálculo dos valores Médios e eficazes e seus fatores de formas

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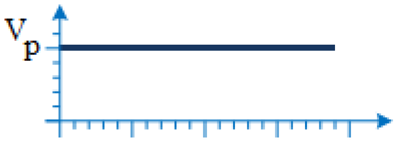
Cálculo dos valores médio e eficazes e fator de forma de várias formas de ondas.

1. Onda contínua.



O valor médio será:

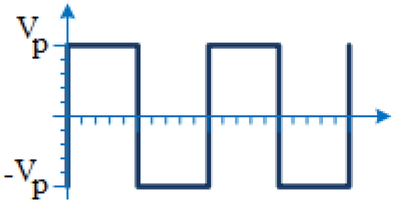
$$f(t)=V_P \Rightarrow \mathbf{V_M = \frac{1}{T}\int_0^T V_P dt = \frac{V_P}{T} t_0^T = \frac{V_P}{T} T = V_P}$$

O valor eficaz será:

$$\mathbf{V_{ef} = \sqrt{\frac{1}{T}\int_0^T V_P^2 dt} = \sqrt{\frac{V_P^2}{T} t_0^T} = \sqrt{\frac{V_P^2}{T} T} = \sqrt{V_P^2} = V_P}$$

O fator de forma $\mathbf{FF = \frac{V_P}{V_P} = 1}$

2. Forma de onda quadrada simétrica.



O valor médio será:

$$f(t)=V_P \Rightarrow \mathbf{V_M = \frac{1}{T}\left[\int_0^{\frac{T}{2}} V_P dt - \int_{\frac{T}{2}}^{T} V_P dt\right] = 0}$$

O valor eficaz será para $0 \leq T \geq T/2$

$$\mathbf{V_{ef\,1} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} V_P^2 dt} = \sqrt{\frac{V_P^2}{T} t_0^{\frac{T}{2}}} = \sqrt{\frac{V_P^2}{T}\frac{T}{2}} = \sqrt{\frac{V_P^2}{2}} = \frac{V_P}{\sqrt{2}}}$$

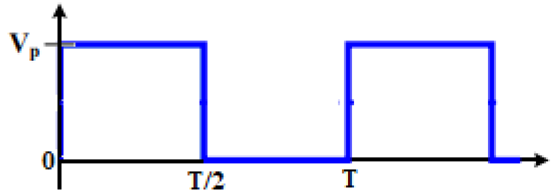
O valor eficaz será para $T/2 \leq T \geq T$

$$\mathbf{V_{ef\,2} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} V_P^2 dt} = \sqrt{\frac{V_P^2}{T} t_0^{\frac{T}{2}}} = \sqrt{\frac{V_P^2}{T}\frac{T}{2}} = \sqrt{\frac{V_P^2}{2}} = \frac{V_P}{\sqrt{2}}}$$

O valor eficaz total será:

$$\mathbf{V_{total}} = \sqrt{\mathbf{V_{ef\,1}^2 + V_{ef\,2}^2}} = \sqrt{\frac{\mathbf{V_P^2}}{\mathbf{2}} + \frac{\mathbf{V_P^2}}{\mathbf{2}}} = \mathbf{V_P}$$

3. Onda quadrada unipolar positiva



O valor médio será:

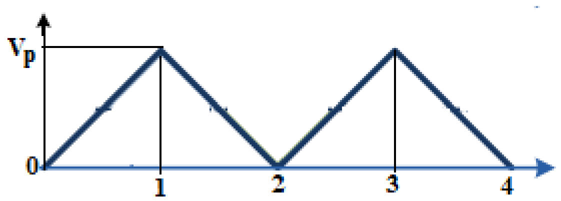
$$\mathbf{V_M} = \frac{\mathbf{1}}{\mathbf{T}}\left[\int_{0}^{\frac{T}{2}} \mathbf{V_P dt}\right] = \frac{\mathbf{V_P}}{\mathbf{T}} \mathbf{t}_0^{\frac{T}{2}} = \frac{\mathbf{V_P}}{\mathbf{T}}\frac{\mathbf{T}}{\mathbf{2}} = \frac{\mathbf{V_P}}{\mathbf{2}}$$

O valor eficaz será:

$$\mathbf{V_{ef}} = \sqrt{\frac{\mathbf{1}}{\mathbf{T}}\int_{0}^{\frac{T}{2}} \mathbf{V_P^2 dt}} = \sqrt{\frac{\mathbf{V_P^2}}{\mathbf{T}} \mathbf{t}_0^{\frac{T}{2}}} = \sqrt{\frac{\mathbf{V_P^2}}{\mathbf{T}}\frac{\mathbf{T}}{\mathbf{2}}} = \sqrt{\frac{\mathbf{V_P^2}}{\mathbf{2}}} = \frac{\mathbf{V_P}}{\sqrt{\mathbf{2}}}$$

O fator de forma $\mathbf{FF} = \dfrac{\dfrac{\mathbf{V_P}}{\sqrt{\mathbf{2}}}}{\dfrac{\mathbf{V_P}}{\mathbf{2}}} = \sqrt{\mathbf{2}}$

4. Onda triangular



O valor médio será:

$f_1(t) = t$ para o intervalo $0 \leq t \leq 1$
$f_2(t) = -t + 2$ para o intervalo $1 \leq t \leq 2$

$$\mathbf{V_M = \frac{1}{2}\int_0^1 V_P t dt + \frac{1}{2}\int_1^2 V_P(-t+2)dt = \frac{V_P}{2}[\int_0^1 tdt - \int_1^2 tdt + \int_1^2 2dt]}$$

$$\mathbf{V_M = \frac{V_P}{2}[(\frac{t^2}{2})_0^1 - (\frac{t^2}{2})_1^2 + 2t_1^2 = \frac{V_P}{2}[\frac{1}{2} - \frac{3}{2} + 2] = \frac{V_P}{2}}$$

O valor eficaz para $0 \leq t \leq 1$

$$\mathbf{V_{ef\,1} = \sqrt{\frac{1}{2}\int_0^1 V_P^2 t^2 dt} = \sqrt{\frac{V_P^2}{2}\frac{t^3}{3}\Big|_0^1} = \sqrt{\frac{V_P^2}{6}} = \frac{V_P}{\sqrt{6}}}$$

O valor eficaz para $1 \leq t \leq 2$

$$\mathbf{V_{ef\,2} = \sqrt{\frac{1}{2}\int_1^2 V_P^2(-t+2)^2 dt} = \sqrt{\frac{V_P^2}{2}\int_1^2 (t^2 - 4t + 4)dt}}$$

$$\mathbf{V_{ef\,2} = \sqrt{\frac{V_P^2}{2}[\int_1^2 t^2 dt - \int_1^2 4tdt + \int_1^2 4dt]} = \sqrt{\frac{V_P^2}{2}[(\frac{t^3}{3})_1^2 - (\frac{4t^2}{2})_1^2 + 4t_1^2}}$$

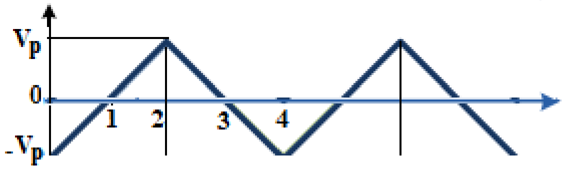
$$\mathbf{V_{ef\,2} = \sqrt{\frac{V_P^2}{2}[7/3 - 6 + 4]} = \sqrt{\frac{V_P^2}{6}} = \frac{V_P}{\sqrt{6}}}$$

O valor eficaz total será:

$$\mathbf{V_T = \sqrt{\frac{V_P^2}{6} + \frac{V_P^2}{6}} = \frac{V_P}{\sqrt{3}}}$$

O fator de forma $\mathbf{FF = \dfrac{\frac{V_P}{\sqrt{3}}}{\frac{V_P}{2}} = \frac{2}{\sqrt{3}}}$

5. Onda triangular simétrica



Para o intervalo $0 \leq t \leq 2$ => $f(t) = V_P\,(t - 1)$
Para o intervalo $2 \leq t \leq 4$ => $f(t) = V_P(-t + 3)$

O valor médio será:

$$\mathbf{V_M = \frac{1}{4}\int_0^2 V_P(t-1)dt + \frac{1}{4}\int_2^4 V_P(-t+3)dt = \frac{V_P}{4}[\int_0^2 tdt - \int_0^2 dt - \int_2^4 tdt + \int_2^4 3dt]}$$

$$\mathbf{V_M = \frac{V_P}{4}[(\frac{t^2}{2})_0^2 - t_0^2 - (\frac{t^2}{2})_2^4 + 3t_2^4] = \frac{V_P}{4}[2-2-6+6=0}$$

Para o intervalo $0 \le t \le 2$

$$\mathbf{V_{ef\,1} = \sqrt{\frac{1}{4}\int_0^2 (V_P(t-1))^2 dt} = \sqrt{\frac{V_P^2}{4}[\int_0^2 t^2 dt - \int_0^2 2tdt + \int_0^2 dt]}}$$

$$\mathbf{V_{ef\,1} = \sqrt{\frac{V_P^2}{4}[\frac{t^3}{3}\Big|_0^2 - \frac{2t^2}{2}\Big|_0^2 + t_0^2} = \sqrt{\frac{V_P^2}{4}[\frac{8}{3} - 4 + 2]} = \sqrt{\frac{V_P^2}{4}\frac{2}{3}} = \frac{\sqrt{V_P}}{\sqrt{6}}}$$

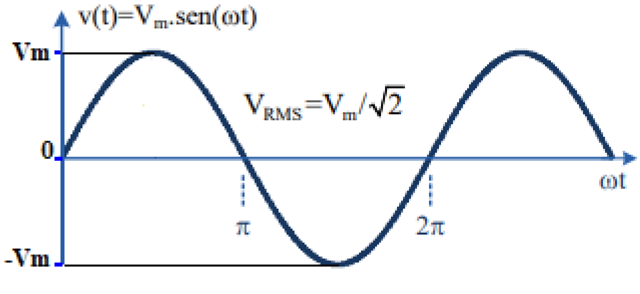
Para o intervalo $2 \le t \le 4$ e analogamente ao exercício anterior

$$\mathbf{V_{ef\,2} = \sqrt{\frac{V_P^2}{6}} = \frac{V_P}{\sqrt{6}}}$$

O valor total

$$\mathbf{V_T = \sqrt{\frac{V_P^2}{6} + \frac{V_P^2}{6}} = \frac{V_P}{\sqrt{3}}}$$

6. Onda senoidal



O valor médio será:

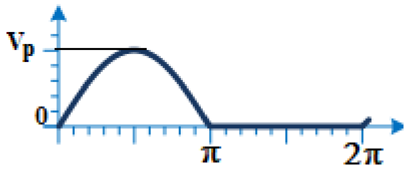
$$\mathbf{V_M = \frac{1}{2\pi}\int_0^{2\pi} V_m sen\omega t dt = \frac{V_m}{2\pi}(-1)\cos\omega t_0^{2\pi} = \frac{V_m}{2\pi}(-1)(\cos 2\pi - \cos 0) = 0}$$

O valor eficaz será:

$$V_{ef} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 sen^2\omega t dt} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}(\frac{1}{2} - \frac{\cos 2\omega t}{2})dt} = \sqrt{\frac{V_m^2}{2\pi}[\int_0^{2\pi}\frac{1}{2}dt - \frac{1}{2}\int_0^{2\pi}\cos 2\omega t dt}$$

$$V_{ef} = \sqrt{\frac{V_m^2}{4\pi}t_0^{2\pi} - 0} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

7. Onda senoidal retificada em meia onda.



O valor médio será:

$$V_M = \frac{1}{2\pi}\int_0^{\pi} V_m sen\omega t dt = \frac{V_m}{2\pi}(-1)\cos\omega t_0^{\pi} = \frac{V_m}{2\pi}(-1)(\cos\pi - \cos 0) = \frac{V_m}{\pi}$$

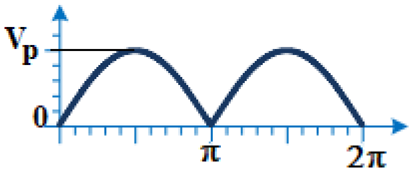
O valor eficaz será:

$$V_{ef} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 sen^2\omega t dt} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}(\frac{1}{2} - \frac{\cos 2\omega t}{2})dt} = \sqrt{\frac{V_m^2}{2\pi}[\int_0^{\pi}\frac{1}{2}dt - \frac{1}{2}\int_0^{\pi}\cos 2\omega t dt}$$

$$V_{ef} = \sqrt{\frac{V_m^2}{4\pi}t_0^{\pi} - 0} = \sqrt{\frac{V_m^2}{4}} = \frac{V_m}{2}$$

O fator de forma $FF = \frac{\frac{V_p}{\sqrt{2}}}{\frac{V_P}{\pi}} = 2,22$

8. Onda senoidal retificada em onda completa



O valor médio será:

$$\mathbf{V_M} = \frac{\mathbf{1}}{\pi}\int_{\mathbf{0}}^{\pi}\mathbf{V_m sen\omega tdt} = \frac{\mathbf{V_m}}{\pi}(-\mathbf{1})\cos\omega\mathbf{t}_{\mathbf{0}}^{\pi} = \frac{\mathbf{V_m}}{\pi}(-\mathbf{1})(\cos\pi - \cos\mathbf{0}) = \frac{\mathbf{2V_m}}{\pi}$$

O valor eficaz será:

$$\mathbf{V_{ef}} = \sqrt{\frac{\mathbf{1}}{\pi}\int_{\mathbf{0}}^{\pi}\mathbf{V_m^2 sen^2\omega tdt}} = \sqrt{\frac{\mathbf{V_m^2}}{\pi}\int_{\mathbf{0}}^{\pi}(\frac{\mathbf{1}}{\mathbf{2}} - \frac{\cos\mathbf{2\omega t}}{\mathbf{2}})\mathbf{dt}} = \sqrt{\frac{\mathbf{V_m^2}}{\pi}[\int_{\mathbf{0}}^{\pi}\frac{\mathbf{1}}{\mathbf{2}}\mathbf{dt} - \frac{\mathbf{1}}{\mathbf{2}}\int_{\mathbf{0}}^{\pi}\cos\mathbf{2\omega tdt}}$$

$$\mathbf{V_{ef}} = \sqrt{\frac{\mathbf{V_m^2}}{\mathbf{2}\pi}\mathbf{t}_{\mathbf{0}}^{\pi} - \mathbf{0}} = \sqrt{\frac{\mathbf{V_m^2}}{\mathbf{2}}} = \frac{\mathbf{V_m}}{\sqrt{\mathbf{2}}}$$

O fator de forma $\mathbf{FF} = \frac{\frac{\mathbf{V_p}}{\sqrt{\mathbf{2}}}}{\frac{\mathbf{2V_P}}{\pi}} = \mathbf{1{,}11}$